**LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034**

**M.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2024**

**PMT2MC04 – COMPLEX ANALYSIS**

Date: 12-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

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| | **SECTION A – K1 (CO1)** |
| | **Answer ALL the questions** **(5 x 1 = 5)** |
| 1. | **Answer the following** |
| a) | Define the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^n$. |
| b) | What are the zeros of an analytic function $f(z)=\frac{(z-2)^3(z+i)^3}{(z-3)^4}$ and mention their multiplicities? |
| c) | What do you mean by a closed rectifiable curve $\gamma_0$ in $G$ homotopic to zero? |
| d) | Define a convex set with an example. |
| e) | State Functional equation. |
| | **SECTION A – K2 (CO1)** |
| | **Answer ALL the questions** **(5 x 1 = 5)** |
| 2. | **Choose the correct answer for the following** |
| a) | The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$<br>(i) 1 (ii) 2 (iii) n (iv) ∞ |
| b) | If $p(z)=z^n+a_{n-1}z^{n-1}+\dots a_0$ then $\lim_{n\to\infty} p(z)=¿¿$<br>(i) 0 (ii) 1 (iii) ∞ (iv) n |
| c) | If $f:G\to C$ is an analytic function and $\gamma$ is a closed rectifiable curve such that $\gamma$ 0 then $\int_{\gamma} f=¿¿$<br>(i) nonzero (ii) 2 (iii) 1 (iv) 0 |
| d) | $E_0(z)=¿$<br>(i)1 (ii) $1-z$ (iii) $1+z$ (iv) $1+2z$ |
| e) | If $\lvert z\rvert\le 1$ and $p\ge 0$ then $\lvert 1-E_p(z)\rvert\le$<br>(i) $\lvert z\rvert^{p-1}$ (ii) $\lvert z\rvert^{p+1}$ (iii) $\lvert z\rvert^{p}$<br>(iv) $\lvert z\rvert^{p+2}$ |
| | **SECTION B – K3 (CO2)** |
| | **Answer any THREE of the following** **(3 x 10 = 30)** |
| 3. | Prove $\int_0^{2\pi} \frac{e^{is}}{e^{is}-z}\,ds=2\pi\, if\, \lvert z\rvert<1$. |
| 4. | State and prove the fundamental theorem of algebra. |
| 5. | Let $\Re z_n>-1$. Prove that the series $\sum \log(1+z_n)$ converges absolutely if and only if the series $\sum z_n$ |

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| | converges absolutely. |
| 6. | Prove that a differentiable function on $[a, b]$ is convex if and only if $f'$ is increasing. |
| 7. | State and prove the Gauss's formula. |

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| | **SECTION C – K4 (CO3)** |
| | **Answer any TWO of the following** **(2 x 12.5 = 25)** |
| 8. | State and prove Cauchy's integral formula and apply to evaluate $\int_{\gamma} \frac{1}{z-a} dz$ where $\gamma = a + \Re^{¿}, 0 \le t \le 2\pi.$ |
| 9. | State and prove Morera's theorem. |
| 10. | State and prove Schwarz's lemma. |
| 11. | If $\gamma_0$ and $\gamma_1$ are two closed rectifiable curves in G such that $\gamma_0 \quad \gamma_1$ explain how $\int_{\gamma_0} f = \int_{\gamma_1} f$. |
| | **SECTION D – K5 (CO4)** |
| | **Answer any ONE of the following** **(1 x 15 = 15)** |
| 12. | State the characterization of gamma function and defend it. |
| 13. | Prove the Weierstrass factorization theorem and evaluate the factorization of sine function. |
| | **SECTION E – K6 (CO5)** |
| | **Answer any ONE of the following** **(1 x 20 = 20)** |
| 14. | Defend Goursat's theorem. |
| 15. | Let $G$ be a simply connected region which is not the plane and let $a \in G$. Construct a unique analytic function $f: G \rightarrow C$ having the properties:<br>(a) $f(a)=0$ and $f'(a)>0$<br>(b) f is one-one<br>(c) $f(G)=\{z:\|z\|<1\}$ |

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